Quasi-Static Analysis of a Leg-Wheel Hybrid Vehicle for Enhancing Stair Climbing Ability

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Abstract
This paper presents quasi-static analysis of a leg-wheel hybrid vehicle for enhancing stair climbing ability. The vehicle consists of four spoke-wheels. The number of spokes for each wheel can be altered by varying the number of strips attaching to the wheel hub. The effects of having different number of spokes are studied in two bases: climbing ability over an obstacle and the required coefficient of friction between spoke tips and stairs. The climbing ability is measured by the maximum height of obstacle that the vehicle is able to travel. For a certain friction coefficient, the limit for obstacle height can also be predicted. Leg-wheel vehicles with four, six and twelve spokes are also built and tested. Results from the quasi-static analysis and the experiments are shown to be comparable. The slight difference is due to deformations of the spoke strips from the vehicle weight. Calculation and experimental results are in agreement. The maximum step height of the leg wheel can roll over in each case is relatively more than its hub. The twelve-spoke wheel gives the maximum absolute height among the three cases.

Keywords: Leg-Wheel Hybrid Vehicle, Stair climbing ability, Quasi-static analysis

1. Introduction
In recent years, a number of developments have been done on robots having different types of locomotion, depending on the environment. While wheel mechanisms are relatively simple and allow quick mobility, leg mechanisms have shown tremendous advantages in climbing over obstacles.

In early studies, for example, Raibert and his team successfully built monopod and biped robots which are able to hop over stairs [1].

PROLERO (PROtotype of LEgged ROver) was developed in 1996. This robot used a total of 6 motors—one for each of its 6 legs. Each leg is comprised of an L-shaped spoke rotating in a full circle to move its body forward and lift it over an obstacle [2].

RHex is a compliant hexapod robot inspired by cockroach locomotion. This robot has the ability to traverse over rough terrain and climb stairs [3],[4],[5].

Whegs™ (Wheel+legs) robots followed the design of RHex as demonstrating the advantages of the combination of wheels and legs. Each wheg consists of a three-spoke configuration. The robot uses one motor for propulsion and is moved in an alternating tripod gait [6].

This study presents a new leg-wheel quadruped hybrid vehicle similar to Whegs™ but with adjustable number of spokes as shown in Fig. 1. Climbing ability of the vehicle with normal wheel and leg wheel with different number of spokes up to twelve is investigated. Method of climbing ability analysis is proposed by using quasi-static assumptions. The analysis involves evaluation of the maximum obstacle height that the vehicle can travel as well as the required coefficient of friction between the spoke tips and obstacles. Experiments for the vehicle with four, six, and twelve spokes are performed in controlled field to obtain climbing ability in each case. Six-spoke wheel is chosen as a representative to be tested in a rescued terrain.
2. Climbing Analysis

Climbing ability of a vehicle over an obstacle is examined by using quasi-static analysis based on two principles: geometric consideration and static friction condition. The former offers the maximum obstacle height that the wheel is able to travel from geometric point of view, provided that the applied torque and the frictional force are sufficient. The latter proposes a method to approximate the step height which the vehicle can climb over for a specified applied torque and friction coefficient between the wheel and the obstacle.

A. Geometric Consideration

Schematic diagrams for step climbing of normal wheel and spoke wheel mechanisms are illustrated in Fig. 2(a) and 2(b), respectively. In this case, climbing ability depends mainly upon the wheel radius. It can be seen from Fig. 2(a) that unless the wheel climbs up vertically, the maximum obstacle height \( h \) is limited to the wheel radius \( r \).

Figure 2. Schematic diagram for (a) normal wheel (b) compliant spoke wheel

Figure 2(b) shows the initial condition (position and orientation) to deliver the highest climbing ability for five-compliant-spoke wheel. The spokes are arranged evenly with an angle of \( \phi \) and the wheel radius \( r \) is measured from the hub center to the tip of the spokes. The angle \( \alpha \) is measured from the vertical line relative to the nearest spoke touching the ground where the clockwise direction gives positive angle. The maximum height of climbing for spoke wheel \( h' \) can then be expressed as

\[
h' = r \left[ \sin \left( \frac{\phi}{2} \right) + \cos \alpha \right] \quad (1)
\]

Note that initial condition plays an important role in climbing ability for spoke wheel model cases. Having different number of spokes produces different initial positions to obtain highest climbing ability. Figure 3 demonstrates the initial conditions yielding the maximum heights for climbing over obstacles for spoke wheels with different number of strips. Three, six and five strips are chosen to represent the angle \( \alpha \) for positive, neutral and negative directions.

Figure 3. Initial conditions for spoke wheel with (a) three (b) six and (c) five strips

B. Static Friction Condition

For a system in which the input torque and the coefficient of friction are specified, they may not satisfy the conditions for maximum geometric climbing ability. For these circumstances, quasi-static analysis is utilized to arrive at the maximum obstacle heights.

Figure 4 shows the force balance conditions at the state where the wheel starts to climb over an obstacle for normal and spoke wheels. In this figure, \( F_1 \) is the horizontal component of force the wheel received from the body, \( N_1 \) is the load from the body weight, \( F_2 \) is the frictional force, \( N_2 \) is the normal force occurring between the wheel and the step, \( T \) is an applied couple from the motor, and \( \mu \) is the static coefficient of friction between the wheel and the step obstacle.

\[
\begin{align*}
T & = F_1 \\
N_1 & = F_2 \\
F_2 & = \mu N_2 \\
N_2 & = T
\end{align*}
\]

Figure 4. Force balance conditions at the climbing state for (a) normal wheel (b) spoke wheel

Takahashi et al. [7] derived the maximum angle \( \theta \) from force balance conditions and a static friction condition in which the wheel starts to climb up as

\[
\theta = \cot^{-1} \left( \frac{1 - \mu \epsilon}{\mu + \epsilon} \right) \quad (2)
\]
where \(0 < \theta \leq 90^\circ\) and \(\varepsilon\) is the ratio of \(F_1\) and \(N_1\). If one knows \(\mu\) and \(\varepsilon\), the maximum step height \(h_s\) for wheel mechanism can be determined. Hence, the wheel can start to climb up the step taller than \(r (\theta > 90^\circ)\) when \(\mu \varepsilon > 1\).

From Fig. 4(b), let \(\gamma\) be the angle between the horizontal line and the tangential line of the spoke tip touching the obstacle’s top surface. The angle \(\gamma\), that establishes the maximum obstacle height for spoke wheel \(h_i'\), can be calculated in a similar manner as

\[
\gamma = \cot^{-1} \left( \frac{1 - \mu \varepsilon}{\mu + \varepsilon} \right)
\]  

(3)

Note that for the spoke wheel case, the maximum climbing angle \(\theta_i = \gamma + 90^\circ\) where \(0 \leq \gamma \leq \phi/2\). As a result, the angle \(\theta_i\) is generally greater than 90 degrees and climbing ability of spoke wheels is automatically higher than their wheel radii. The maximum step height for spoke wheel \(h_i'\) is calculated by

\[
h_i' = r \left( \sin \gamma + \cos \alpha \right)
\]  

(4)

It can be seen from Eqs. (3) and (4) that different coefficients of friction and input thrust forces introduce different initial conditions to obtain the maximum permitted height. The maximum climbing ability is achieved when \(\gamma\) is equal to \(\phi/2\). This condition can be used to compute the required coefficient of friction when the applied force is known and vice versa.

For a particular instance where there is no applied force \((\varepsilon = 0)\), Eq. (3) deduces to \(\mu_0 = \tan \gamma\) which constitutes the static coefficient of friction for a sloped floor of angle \(\gamma\).

The graph in Fig. 5 shows relationships between the ratio of geometric maximum height and the spoke wheel radius \((h'/r)\), the required coefficient of friction to achieve \(h'\) without applied force \((\mu_0)\), the required thrust force ratio to achieve \(h'\) when \(\mu = 0 \ (\varepsilon)\), and the number of spokes. Since the values of \(\mu\) and \(\varepsilon\) in Eq. (3) are interchangeable, they yield analogous curves.

3. Vehicle Design

For simplicity, this vehicle is designed to have spoke-wheel where the number of spokes can be adjusted. The enclosed body allows it to run in outdoor environment. The vehicle body is 24 cm long, 35 cm wide and 10 cm high. The radius \(r\) measured from hub center to spoke tip is 12 cm. Total body weight including 12 V battery is about 7.5 kg. It uses two gear head DC motors 7.2 V model 380K75 made by TAMIYA. The vehicle is controlled by wired joystick. Each motor is driving a pair of spoke wheels on each side via a sprocket chain mechanism with a teeth ratio of 9: 36. The vehicle is also designed to be able to move in an upward position. The battery and motor control are on-board with a wired remote control.

Components of the compliant spoke wheel are depicted in Fig. 6. The wheel consists of an inner hub, an outer hub, and removable spoke strips. The hub is designed such that the steel strips up to twelve strips can be inserted evenly along the hub circumference. The strips are made of 0.7 mm thick cold-rolled steel plates.
DIN1625. The spokes are also coated with synthetic rubber to increase friction when climbing.

4. Experimental Results

The experiment in searching the maximum permitted height of 4, 6 and 12 spokes are performed to compare with the calculation along with Eq. (4). The vehicle was tested on climbing concrete stairs by varying the height of support. Coefficient of friction between the strip and obstacle surface used in the calculation is 0.2 and the ratio \( \varepsilon \) is assumed to be 0.1.

Four-spoke vehicle shown in Fig.7 demonstrates the ability to climb over an obstacle 12 cm high which is less than the expected value of 15.2 cm. This can be the result of the deflection of the strips due to the vehicle weight. It can be observed that the body height of the vehicle measured from the hub center relative to ground is less than the wheel radius. In this case, the body height is 7 cm.

Four-spoke wheel climbing up the stairs

Figure 7. Four-spoke wheel climbing up the stairs

The vehicle with six-spoke wheels can climb over 13 cm which is higher than its hub as shown in Fig. 8. For the case of six-spoke wheel, body height of the vehicle is about 9.9 cm. Twelve spokes can climb an obstacle height of 13.5 cm shown in Fig. 9 whereas the body height is about 11.8 cm. Noting that experimental climbing ability is quite steady with the number of spokes equal to six.

Figure 8. Six-spoke wheel climbing up the stairs

Figure 9. Twelve-spoke wheel climbing up the stairs

Comparison of climbing ability between the calculated values from Eq. (4) and experimental results are shown in Fig 10. The values from both methods show the same tendency in which climbing ability increases when number of spokes increases. For all the three cases studied, the values from experiments are less than the calculated values. The difference between the two values is the largest for four-spoke wheel which shows substantial deflections of spoke strips. Better approximations of climbing ability for compliant spoke wheels can be achieved by considering the effects of the vehicle weight to the deformation of the spoke strips.

Figure 10. Experimental results on the climbing ability of 4, 6 and 12 spokes compared to calculated values

Figure 11 shows the vehicle is climbing over a specially constructed disaster site of Robocup Rescue league in Thailand. In this case, the obstacle is made of wood. Six compliant spokes is chosen to test ride over the site and it can pass over the site without difficulty.
5. Discussions

In this research, we proposed a method of climbing analysis for spoke wheel mechanism based on quasi-static principles. A spoke-wheel hybrid vehicle for enhancing stair climbing ability is successfully built to study the advantages of climbing ability. The number of spokes can be adjusted by changing the number of strips attaching to the wheel hub to obtain the optimum climbing ability. Theoretically, few spokes have the ability in climbing higher level of obstacle but the flexible spokes lower the body height. This will result in decreasing climbing ability. The less number of spokes also require high coefficient of friction. The availability of the proposed analysis is confirmed by experimental results.

However, many spokes with compliant parts take advantages in the use of lower coefficient of friction and climbing ability over obstacle. The experimental results showed that all four, six and twelve spokes with compliance can climb over than their hub radius whereas twelve spokes give the maximum height of climbing among these three cases. Getting more spokes such as six and twelve can have more supported legs to distribute the same load. It also lifts the body high and is able to climb over higher level of obstacle. The results also showed that climbing ability of six spokes is slightly less than twelve spokes. Heuristically, six spokes consume less amount of power than twelve spokes, then it should be more practical in traversing over various terrain. In addition, the construction of six spokes is simpler than twelve spokes.

6. References