Influence of Roughness on Two Surfaces under Line Contact in EHL with Non-Newtonian Fluid

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Abstract

In this paper, the effect of surface roughness and elastic deformation of surfaces in line contact under elastohydrodynamic lubrication are presented. A power law model is developed for the elastohydrodynamic lubrication regime. The modified Reynolds equations with elasticity equation were formulated for the two surfaces under heavy load conditions. Multigrid, multilevel techniques and Newton’s method were used to calculate the elastohydrodynamic bearing to obtain the oil film pressure profiles, oil film thickness profile, load capacity, attitude angle and friction coefficient at various surface roughness parameters. The results show that the surface roughness and elastic deformation significantly affect the static characteristics of bearings under severe operating conditions.

Keywords: Modified Reynolds equation, Elasticity equation, Elastohydrodynamic lubrication, Newton Raphson method.

NOMENCLATURE

\( E = \) Equivalent modulus of elasticity (Pa)
\( E_i = \) Modulus of elasticity of Roller (Pa)
\( E_2 = \) Modulus of elasticity of Plate (Pa)
\( h = \) Film thickness (m)
\( X = \) dimensionless x coordinate
\( H = \) dimensionless film thickness at \( X = 0 \)
\( k_1 = \) Thermal conductivity of Roller \( W/(m-K) \)
\( k_2 = \) Thermal conductivity of Plate \( W/(m-K) \)
\( K = \frac{\pi^2U}{16(W')^2} \)
\( n = \) Power law index
\( m = \) Apparent viscosity at the shear rate of unit
\( p = \) Pressure (Pa)
\( p_H = \) Maximum Hertzian pressure (Pa) = \( E \left( \frac{W'}{2\pi} \right) \)
\( P = \) Dimensionless pressure
\( R = \) Equivalent radius (m)
\( t = \) Time (s)
\( t^* = \) Dimensionless time
\( \rho = \) dimensionless density
\( u^* = \) Dimensionless velocity
\( \overline{u} = \) Mean velocity (m/s) = \( \frac{u_2 + u_1}{2} \)
\( W_f = \) Hydrodynamic load (N)
\[ \varepsilon = \frac{\rho H^3}{\mu} \]

\[ K = \text{Constant} = \frac{3\pi^2 U}{4W^2} \]

\[ W = \text{Dimension load parameter} = \frac{W}{ER} \]

\[ U = \text{Dimension speed parameter} = \frac{U_0}{ER} \]

1. Introduction

Since the numerical solution of elastohydrodynamic lubrication (EHL) problems were solved by Dowson and Higginson [8] in 1959. Many numerical analyses have been obtained in the area ranging from TEHD lubrication problems to transient EHL problems. Lee, R. and Hamrock, B.J. [7] used Newton-Raphson method to calculate time dependent EHL problems under low load conditions. In 1990, Khonsari, M. M., Wang, H. S. and Qi, Y.L. [3] have formulated Reynolds and energy equations for non-Newtonian liquid-solid lubricants in line contact. The solid lubricant had significant effects in raising the film thickness, load capacity, temperature and friction coefficient in the full EHL regime. Lubrecht, A.A., ten Napel, W.E. and Bosma, R. [6], they presented that the multigrid algorithm has more efficient than Newton-Raphson method in solving EHL with roughness effect. However, the multi-grid technique has been developed to solve transient thermoelastohydrodynamic lubrication (TEHL) by Osborn, F. K. and Sadeghi, F. [1]. The load and speed were significant factors that affect the response of lubricated line contact. The time to reach steady-state position was a very strong function of speed. Ai, X., Cheng, H. S. [4] presented the formulation of transient rough EHL problem using multi-grid technique. The results showed that surface roughness induced transient effects significantly on pressure distribution in line contact.

Recently, a multigrid multilevel algorithm has been developed to determine three-dimensional minimum film thickness and three-dimensional maximum pressure in EHD line contact by Franscisco A., Frene, J., and Blonin, A. [9]. The results on two-dimensional minimum film thickness were overestimated when compared to a three-dimensional analysis. In this study, the numerical calculation of transient TEHL with non-Newtonian liquid-solid lubricants under heavy load change was presented. Finite difference method multigrid with full approximate scheme techniques and Newton’s method were implemented to calculate the transient TEHL under a load change.

2. Theoretical analysis

Figure 1 Schematic diagram of a cylindrical roller on Flat Pate under EHL

Time dependent elastohydrodynamic lubrication of rolling/sliding line contact can be solved simultaneously by using the Reynolds and elasticity equation to obtain pressure temperature and film thickness distribution
Reynolds Equation

The relationship between shear stress and shear rate of non-Newtonian lubricant in this work can be approximated using power-law model as:

\[ \tau_{xy} = \mu \frac{\partial u}{\partial y}, \quad \tau_{zy} = \mu \frac{\partial w}{\partial y} \]  

(1)

where the equivalent viscosity

\[ \mu^* = m_0 \left[ \left( \frac{\partial \mu}{\partial y} \right)^2 + \left( \frac{\partial \mu}{\partial y} \right)^2 \right]^{\frac{n-1}{2}} \]  

(2)

\[ n = \text{Power law index} \]

\[ m_0 = \text{Apparent viscosity at the unity shear rate} \]

In this study, for low particle concentration in the liquid-solid lubricants, the single phase flow can be simplified in this work. The integral of the momentum and continuity equations, the time dependent Reynolds equations with non-Newtonian lubricants for infinitely long cylindrical roller can be obtained as:

\[ \frac{\partial}{\partial X} \left[ \bar{\rho} H^2 \left( \frac{1}{\bar{\mu} e_2} - \frac{\bar{\mu} e_0}{\bar{\mu} e_1} \right) \frac{\partial P}{\partial X} \right] = K \frac{\partial}{\partial X} (\bar{\rho} H) \]

\[ + K S \frac{\partial}{\partial X} \left[ \bar{\rho} H \left( 1 - 2 \frac{\bar{\mu} e_0}{\bar{\mu} e_1} \right) \right] \]

(3)

where

\[ \frac{1}{\bar{\mu} e_{i,j}} = \int Y^j \frac{dY}{\bar{\mu}} \]

Where the boundary conditions are

\[ X = X_{\text{inlet}}; \quad P = 0 \]

\[ X = X_{\text{exit}}; \quad P = \frac{dP}{dX} = 0 \]  

(4)

The apparent viscosity in the power-law model needs to be included as a correction factor for viscosity temperature-pressure correction factor and the correction factor for solid particles in the lubricants according to Rylander [5]. The dimensionless apparent viscosity can be written as:

\[ \bar{\mu}^* = m_0 \left[ \frac{\bar{\mu} \pi}{8 R W H} \right]^{\frac{n-1}{2}} \left[ \frac{\partial u^*}{\partial Y} \right]^{n-1} \exp \left[ \ln (\mu_0) + 9.67 \left( -1 + \left( 1 + 5.1 \times 10^{-9} \rho P \right)^6 \right) \right] \]

(5)

The dimensionless density of the liquid–solid lubricant according to Dawson, Higginson obeys the following relation

\[ \bar{\rho} = 1 + \frac{0.6 \times 10^{-9} \rho P}{1 + 1.7 \times 10^{-9} \rho P} \]  

(6)

The film thickness can be expressed due to the deformation of the surfaces as:

\[ H = H_o + \frac{X^2}{2} - \frac{1}{\pi} \int_{X_{\text{in}}}^{X_{\text{out}}} P(\xi) \ln |X - \xi| d\xi \]

(7)

Load Carrying Capacity

The total load carrying capacity from the hydrodynamic action \( w_f \)

\[ w_f = \int_{\text{inlet}}^{\text{exit}} p(x) dx \]  

(8)

3. Numerical Solution

The simultaneous systems of time-dependent Reynolds and elasticity and energy equations of the elastohydrodynamic lubrication with non-Newtonian were calculated using Newton Raphson and multigrid with full approximation scheme techniques. The time dependent Reynolds equation can be discretized to obtain
the fully implicit scheme as:

\[
\begin{align*}
\rho_i & = \rho_i^{n+1} + \frac{1}{2} \Delta t \left[ (\nu_i^{n+1} - \nu_i^{n}) + (\nu_i^{n+1} - \nu_i^{n}) \right] \\
\mu_i & = \mu_i^{n+1} + \frac{1}{2} \Delta t \left[ (\nu_i^{n+1} - \nu_i^{n}) + (\nu_i^{n+1} - \nu_i^{n}) \right] \\
\end{align*}
\]

\[
\begin{align*}
\rho_i & = \frac{1}{2} \Delta t \left[ (\nu_i^{n+1} - \nu_i^{n}) + (\nu_i^{n+1} - \nu_i^{n}) \right] \\
\mu_i & = \frac{1}{2} \Delta t \left[ (\nu_i^{n+1} - \nu_i^{n}) + (\nu_i^{n+1} - \nu_i^{n}) \right] \\
\end{align*}
\]

\( i = 1, 2 \)

(9)

Where

\[
\begin{align*}
\epsilon_i & = \beta_i H_i^3 \left( \frac{1}{\mu_{i,2}^{n+1}} - \frac{\mu_{i,0}^{n+1}}{\mu_{i,1}^{n+1}} \right) \\
\end{align*}
\]

\[
\begin{align*}
\epsilon_{i+\frac{1}{2}} & = \left( \frac{\epsilon_i + \epsilon_{i+1}}{2} \right) \\
\end{align*}
\]

\[
\begin{align*}
\epsilon_{i-\frac{1}{2}} & = \left( \frac{\epsilon_i + \epsilon_{i-1}}{2} \right) \\
\end{align*}
\]

\( i = 1, 2 \)

(10)

During each time interval, the Reynolds elasticity and energy equations are calculated using boundary conditions and initial conditions in equation (5),(8) and (10) to obtain pressure distribution. In this numerical calculation the film thickness between the surfaces of the roller is an infinitely long on flat surface.

Table 1. Material property of Roller

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equivalent radius, m. : R</td>
<td>0.05</td>
</tr>
<tr>
<td>Density, kg − m⁻³ : ( \rho_0 )</td>
<td>8000</td>
</tr>
<tr>
<td>Thermal conductivity, Wm⁻¹K⁻¹ : ( K_1 )</td>
<td>52</td>
</tr>
<tr>
<td>Specific heat, J kg⁻¹K⁻¹ : ( C_{p,1} )</td>
<td>460</td>
</tr>
<tr>
<td>Elastic modulus, Pa. : ( E_1 )</td>
<td>1.8x10¹¹, 2.0x10¹¹, 2.2x10¹¹</td>
</tr>
</tbody>
</table>

Table 2. Material property of Plate

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density, kg − m⁻³ : ( \rho_0 )</td>
<td>8000</td>
</tr>
<tr>
<td>Thermal conductivity, Wm⁻¹K⁻¹ : ( K_2 )</td>
<td>52</td>
</tr>
<tr>
<td>Specific heat, J kg⁻¹K⁻¹ : ( C_{p,2} )</td>
<td>460</td>
</tr>
<tr>
<td>Elastic modulus, Pa. : ( E_2 )</td>
<td>1.8x10¹¹, 2.0x10¹¹, 2.2x10¹¹</td>
</tr>
</tbody>
</table>

4. Result and Discussion

Figure 2 Steady state pressure profiles of Smooth roller and smooth flat plate

\( U = 4 \times 10^{-11}, SAE40, S = 0. \)

Figure 3 Transient film thickness profiles of Smooth roller and smooth flat plate

\( U = 4 \times 10^{-11}, SAE40, S = 0. \)
Figure 4 Steady state pressure profiles of Rough surface roller and smooth flat plate

\[ U = 4 \times 10^{-11}, SAE40, S = 0. \]

Figure 5 Transient film thickness profiles of Rough surface roller and smooth flat plate

\[ U = 4 \times 10^{-11}, SAE40, S = 0. \]

Figure 6 Steady state pressure profiles of Smooth roller and smooth flat plate

\[ U = 2 \times 10^{-11}, W = 5 \times 10^{-5}, SAE40 \]

Figure 7 Transient film thickness profiles of Smooth roller and smooth flat plate

\[ U = 2 \times 10^{-11}, W = 5 \times 10^{-5}, SAE40 \]

Figure 8 Steady state pressure profiles of Rough surface roller and smooth flat plate

\[ U = 2 \times 10^{-11}, W = 5 \times 10^{-5}, SAE40 \]

Figure 9 Transient film thickness profiles of Rough surface roller and smooth flat plate

\[ U = 2 \times 10^{-11}, W = 5 \times 10^{-5}, SAE40 \]
Influence of loads and material elastic on EHD characteristics

In this research work, the elastohydrodynamic lubrication line contact with non-Newtonian lubricants under the dimensionless loads changes from $6 \times 10^{-3}$ to $1 \times 10^{-4}$ at the dimensionless speed parameter, $U = 4 \times 10^{-11}$ and the dimensionless elastics changes from $1.8 \times 10^{-11}$ to $2.2 \times 10^{-11}$ at the dimensionless speed parameter, $U = 2 \times 10^{-11}$ were investigated.

Figures 2 and 3 represent the elastohydrodynamic characteristics of smooth roller surface and smooth flat plate under load change. It is found that pressure is extremely high and film thickness become very thin. Under High load conditions, the equilibrium state can reach faster than the equilibrium state at low load conditions.

Figures 4 and 5 represent the elastohydrodynamic characteristics of rough surface roller on smooth flat plate with various loads conditions. It is found that the roughness surface of roller has significant effects on film pressure profile and the film thickness becomes very thin. Both pressure and film thicknesses obtained are similar to that for smooth surface condition.

Figures 6 and 7 represent the elastohydrodynamic characteristics of smooth roller surface on smooth flat plate with various material elastic moduli. It is found that pressure becomes insignificant difference at the center but the film pressure spike at trialing edge and the pressure decrease for soft material. The minimum film thickness increases for large elastic moduli material.

Figures 8 and 9 represent the elastohydrodynamic characteristics of rough roller surface and smooth flat plate with various elastic moduli. It is found that the roughness has slightly effect to pressure but film thickness become increasing for large elastic moduli material. The minimum film thickness is also higher than the minimum film thickness of the smooth surface.

5. Conclusion

In this research work, the characteristics of Elastohydrodynamic lubrication under severe operating conditions show that the surface roughness has significantly affected to the static characteristics of bearings.

1. The pressure is drop for roughness surface of roller.
2. The film thickness between the rough roller surface and the smooth flat is slightly larger than that for smooth surface roller and smooth plate.

6. Acknowledge

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7. Reference


